

A unified approach to numerical homogenization of flow parameters in porous media

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The aim of upscaling is to determine equivalent homogeneous parameters at a coarse-scale from a spatially oscillating fine-scale parameter distribution. A large variety of upscaling methods is available, among which the homogenization method plays an important role.

This paper presents an extension of the classical homogenization method to nonlinear problems as they occur while upscaling parameters of incompressible, immiscible two-phase flow and while dealing with flow in near well regions.

In all cases homogenization is based on fine-scale steady-state flow equations with periodic boundary conditions. These equations are solved with the finite element method. Results of numerical experiments are given.

1. Introduction

The aim of upscaling is to determine equivalent homogeneous flow parameters at a coarse-scale from a spatially oscillating fine-scale parameter distribution, in such a way that the average behavior of the heterogeneous porous medium is preserved. The spatial fine-scale permeability distribution is generally obtained from geological and geostatistical models. Upscaling then enables the use of a limited number of relatively large grid-blocks in numerical groundwater models or oil reservoir simulators and, at the same time, still allows to account for effects that are due to the fine-scale heterogeneities.

Upscaling of single-phase flow permeability is routinely performed nowadays, cf. (Renard and de Marsily, 1997) and (Wan and Gómez-Hernandez, 1996), and a large variety of methods is available, among which the homogenization method plays an important role. Homogenization has proved its effectiveness for linear problems as they occur in single-phase porous media flow. In spite of some theoretical limitations its range of applicability can be made much wider and cover a class of nonlinear problems too.

Our approach is based on the principle of *preservation of form* stating that the mathematical form of the fine-scale equations should be preserved *as much as possible* on the coarse scale. This principle allows to extend the classical homogenization approach to nonlinear problems. In this paper a unified, homogenization-based approach to evaluate effective flow parameters is presented. We start with discussing one-phase flow. Then it is shown how the method of upscaling of one-phase flow can be extended to a case of a near-well region. Finally, the extension to the upscaling of two-phase flow is given.

The presented methods may be classified as belonging to a class of process-independent methods. In general, the process-independent approach is a way of upscaling in which the coarse-scale results obtained for a particular reservoir can be used in any simulation,

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irrespective of the scenario that is simulated. Process-independent coarse-scale functions can therefore be stored in databases to be retrieved for different types of study performed on the same aquifer or reservoir.

An important consequence of upscaling is the emergence of anisotropy. Even if the heterogeneous fine-scale permeability $\mathbf{k}(x,y,z)$ is isotropic, the upscaled permeability is a tensor \mathbf{K} , represented by the matrix of components

$$\mathbf{K} = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{pmatrix},$$

with the off-diagonal entries possibly not equal zero. This remark is important, as many commonly used simulators cannot handle the off-diagonal components of the permeability tensor.

As an illustration, results of numerical experiments are given.

2. One-phase flow

Considerable work has been devoted to the problem of upscaling permeabilities for single-phase flow, see for instance the reviews by (Renard, de Marsily, 1997), (Wan, Gómez-Hernandez, 1996) and the references therein. The techniques commonly applied for upscaling range from simple averaging of heterogeneous values within a block to methods involving the solution of the flow equation at a fine-scale. The method we are dealing with is *homogenization*. Homogenization is an upscaling method that has been developed especially for periodic media, thus the approach is valid only if the domain is assumed periodic.

Most works on homogenization theory base their presentations on rigorous mathematics, see for instance (Bensoussan, 1978). However, it is possible to base homogenization on a practical and physically meaningful point of view (still yielding the same results), starting from the point that the fine-scale expressions for outflow minus inflow of fluid in a closed volume should be preserved on the coarse scale. This way the basis of the method of homogenization is to equate the fine-scale differential operator to the upscaled differential operator that describes the outflow minus the inflow of fluid in a volume element (Zijl, Trykozko, 2001b), what is expressed as:

$$\nabla \cdot (\mathbf{K} \cdot P(\underline{x})) = \nabla \cdot (\mathbf{k}(\underline{x}) \cdot \nabla p(\underline{x})), \quad (1)$$

where \mathbf{K} (constant) stands for the upscaled permeability, $P(\underline{x})$ denotes the upscaled pressure (linear), $\mathbf{k}(\underline{x})$ is fine-scale permeability and $p(\underline{x})$ is the pressure at the fine-scale. By \underline{x} we denote a point in a three dimensional space, $\underline{x}=(x, y, z)=(x_1, x_2, x_3)$.

Substituting constant \mathbf{K} and linear P into the left hand side of the equation yields the steady-state flow equation:

$$\nabla \cdot (\mathbf{k}(\underline{x}) \cdot \nabla p(\underline{x})) = 0. \quad (2)$$

The fact that the time does not appear in this equation makes the upscaled mobility time-independent.

In order to compute the coarse-scale permeability, the *cell problem* needs to be solved in a periodicity domain. It consists in solving three similar elliptic flow problems (2) with periodic boundary conditions for the three functions $\chi_i = p_i + H_i \cdot x_i$, $i=1, \dots, 3$, where H_i are linearly independent constant vectors. These problems have unique solutions p_i , $i=1, \dots, 3$ defined up to arbitrary constants.

A convenient choice of H_i is $H_{ij} = \delta_{ij}$, where H_{ij} denotes a component j of a vector H_i , yielding the orthonormal boundary conditions $\chi_i = p_i + x_i$, $i=1,\dots,3$, is periodic on the boundary of the periodicity cell.

Then, based on the solutions of (2), the coarse-scale permeabilities \mathbf{K} are derived from the following formulas:

$$K_{ij} = \langle \mathbf{k} \cdot \text{grad } p_j \rangle_i \quad (\text{component } i \text{ of a solution of problem } j) \quad (3a)$$

and/or

$$K_{ij} = \langle \text{grad } p_i \cdot \mathbf{k} \cdot \text{grad } p_j \rangle, \quad (3b)$$

where the brackets $\langle \cdot \rangle$ denote volume averaging. Expression (3a) represents ‘pressure-flux’ averaging to determine \mathbf{K} from the coarse-scale Darcy’s law while expression (3b) corresponds to the coarse-scale dissipation equation and is a special case of ‘pressure-dissipation’ averaging. Any other choice of the ‘constant-jump’ pressure boundary conditions leads to the same homogenized permeability, however with more complex expressions for K_{ij} . If the porous medium is periodic the two averaging approaches, (3a) and (3b) yields the same result, (Zijl, Trykozko, 2001b).

The *computational procedure* reads as follows:

- solve the three elliptic cell-problems;
- compute the components K_{ij} , $i, j = 1,\dots,3$ of tensor \mathbf{K} of the upscaled permeability following the formulas (3a) or (3b).

Thus from the computational point of view, the most important task is to accurately solve the pressure equations.

3. One-phase flow in a near well region

While considering flow in a near well region we assume the homogenization cell is a *circular cylinder*. Again, the starting point of the reasoning is that the fine-scale expressions for the outflow minus the inflow of fluid in a closed volume is preserved on the coarse scale. For that purpose a circular cylindrical homogenization cell around a well is transformed to an equivalent rectangular volume by the introduction of circular cylinder coordinates r, θ, z . In addition, multiplication or, respectively, division with the coefficient r transforms the permeability components:

$$\begin{aligned} \frac{\partial}{\partial r} \left(rk_{11} \frac{\partial p}{\partial r} + k_{12} \frac{\partial p}{\partial \theta} + rk_{13} \frac{\partial p}{\partial z} \right) + \frac{\partial}{\partial \theta} \left(k_{21} \frac{\partial p}{\partial r} + k_{22} / r \frac{\partial p}{\partial \theta} + k_{23} \frac{\partial p}{\partial z} \right) + \\ \frac{\partial}{\partial z} \left(rk_{31} \frac{\partial p}{\partial r} + k_{32} \frac{\partial p}{\partial \theta} + rk_{33} \frac{\partial p}{\partial z} \right) = 0 \end{aligned} \quad (4)$$

This way the fine-scale equations written in r, θ, z coordinates have the same form as (2). Direct use of an original cylindrical cell leads to a number of inconsistencies, whereas the results obtained after the transformation are consistent, as discussed in (Zijl, Trykozko, 2001a).

In this way a new problem of upscaling transformed fine-scale permeability $\tilde{k}(\underline{x})$ has been obtained. This problem may be solved as described in Section 2, yielding a *transformed* coarse-scale permeability $\tilde{\mathbf{K}}$. Now a back transform is needed, aiming to find the coarse-scale permeability \mathbf{K} belonging to the original coordinate system. The back transform consists in

finding the *homogeneous fine-scale* permeability k_{ij}^c that gives the transformed coarse-scale permeability $\tilde{\mathbf{K}}^c$ equal to the coarse-scale permeability $\tilde{\mathbf{K}}$. This homogeneous permeability is then considered as the coarse-scale permeability around the well. The back transform is equivalent to solving the following system of nonlinear equations:

$$\tilde{K}_{ij}^c(k_{11}^c, k_{12}^c, \dots, k_{32}^c, k_{33}^c) - \tilde{K}_{ij} = 0, \quad i, j=1, \dots, 3. \quad (5)$$

Even if the porous medium is periodic, upscaling domains that are bound by curvilinear coordinate planes, for instance upscaling domains bound by cylindrical coordinate, cannot be periodicity domains. Hence, the limitation to periodic upscaling domains is relaxed when dealing with upscaling around wells.

However, still the homogenization around a well may be considered as a reasonable approximation. If more accuracy is required, global upscaling seems to be a reasonable, though much more computationally expensive approach (Zijl, Trykozko, 2001a).

The *computational procedure* for evaluating the coarse-scale permeability in a near-well region consists of the following steps:

- solve the three elliptic cell-problems (4) in a transformed domain with transformed values of the fine-scale permeabilities.
- compute the components of the tensor of the transformed upscaled permeability following the formulas (3a) or (3b).
- perform the back transform by solving the nonlinear system (5).

Our approach does not provide us with a direct tool of dealing with near-well region problems defined originally in a rectangular domain – as is discussed for example in (Mascarenhas and Durlofsky, 2000). Transforming the periodicity cell results in a transformation of the heterogeneity pattern too. Thus, paradoxically, the price to be paid for a uniform approach is a lack of uniformity of problems we deal with.

4. Two-phase flow

An extension to process-independent homogenization of two-phase flow is also possible. Process-independent upscaling means that the coarse-scale relative mobility and capillary pressure functions – that are functions of the coarse-scale saturations – can be used for any simulation, i.e., for all boundary conditions and production histories. Hence, the coarse-scale relative permeability and capillary functions can be stored in a database filled with parameters characterizing the reservoir on different scales. This process-independent approach complements the “dynamic upscaling” approach, see (Barker and Dupouy, 1999) for a review. Numerical homogenization in case of an incompressible, immiscible two-phase flow consists in finding the $P^c - S - K^r$ curves defining the relationships among capillary pressure P^c and coarse-scale mobilities K^r as functions of the large-scale saturation S . The method we present is process-independent and is based on steady-state flow equations with periodic boundary conditions for the capillary pressure. Thus it may be considered as another extension of the method for the numerical homogenization of the absolute permeability as presented in Section 2.

Though much work has been done in the area of upscaling of constitutive relations for multiphase flow, the often repeated conclusion is that there is a need of developing general approaches for the upscaling of $P^c - S - K^r$ relations (Ataie-Ashatiani *et al.*, 2001, 2002).

Let us consider incompressible two-phase flow. Phase 1 is the wetting phase (water) and phase 2 is the non-wetting phase (oil). The continuity equations for incompressible flow are (Helmig, 1997)

$$\phi \frac{\partial s_\alpha}{\partial t} + \nabla \cdot \underline{q}_\alpha = 0, \quad \alpha = 1, 2, \quad (6)$$

where ϕ is the porosity, \underline{q}_α are the fluxes and s_α , $s_1 + s_2 = 1$, are the saturations of water and oil respectively. As in previous cases, the requirement $\langle \nabla \cdot \underline{q}_\alpha \rangle = \nabla \cdot \langle \underline{q}_\alpha \rangle$ is satisfied if the brackets $\langle f \rangle$ denote the volume average $(\Delta x \Delta y \Delta z)^{-1} \int_{\underline{x}-\Delta \underline{x}/2}^{\underline{x}+\Delta \underline{x}/2} f(\underline{x}') d\underline{x}'$ of a function $f(\underline{x})$ over a rectangular cell with volume $\Delta x \Delta y \Delta z$.

The following additional requirement is defined, stating that the *fine-scale* formulation expressing the net volumetric outflow rate of a closed volume, $\nabla \cdot \underline{q}_\alpha$, should have the same form, $\nabla \cdot \langle \underline{q}_\alpha \rangle$, on the *coarse scale*. In the form of an equation, this requirement is written as

$$\nabla \cdot \underline{q}_\alpha = \nabla \cdot \langle \underline{q}_\alpha \rangle, \quad (7)$$

where one can easily recognize the relationship (1).

Requirement (7) is used in two different ways. Firstly, it is applied to prove that Onsager's well-known expression for the dissipations on the fine-scale is 'conserved' on the coarse scale, see (Zijl, Trykozko, 2002) for more details. Secondly, this requirement is used to compute the fine-scale pressure and flux solution.

Muskat's extension of Darcy's law yields the following equations (Helmig, 1997)

$$\underline{q}_\alpha = -\lambda_\alpha \cdot \nabla p_\alpha, \quad \alpha = 1, 2 \quad (8)$$

Here λ_α are the mobility tensors of the two phases, $p_\alpha = \psi_\alpha - \rho_\alpha \underline{g} \cdot \underline{x}$ are the excess pressures, ψ_α are the phase pressures, ρ_α are the phase densities, and \underline{g} is the gravitational acceleration. A generalization of Darcy's laws (8) including terms that couple the water and oil Darcy's laws to each other is not considered.

It is generally accepted that the mobilities are functions of the saturation, *i.e.*, $\lambda_\alpha = \lambda_\alpha(\underline{x}, s_\alpha)$.

The mobilities are generally represented as $\lambda_\alpha(\underline{x}, s_\alpha) = \mu_\alpha^{-1} r_\alpha(\underline{x}, s_\alpha) \cdot \mathbf{k}(\underline{x})$, where $\mathbf{k}(\underline{x})$ is the absolute permeability, and $r_\alpha(\underline{x}, s_\alpha)$ and μ_α are respectively the relative permeabilities and viscosities of the two phases. The absolute permeability $\mathbf{k}(\underline{x})$ is fluid independent whereas the relative permeabilities $r_\alpha(\underline{x}, s_\alpha)$, which are assumed scalars, can be considered as scaling factors which depend on the saturation of the phases. Also the capillary pressure $\psi = \psi_2 - \psi_1$ is assumed to be a function of one of the saturations, *i.e.*, $\psi = \psi(\underline{x}, s_1)$.

Substitution of Darcy's laws (8) into the continuity equations (7) yields two Boussinesq equations for the two excess pressures

$$\nabla \cdot (\lambda_\alpha \cdot \nabla p_\alpha) = 0, \quad \alpha = 1, 2 \quad (9)$$

For moderate viscosity ratios such a steady state model is well suited. Again, the formal similarity to Equation (2) is obvious.

Equations (9) have to be solved in a homogenization cell with periodic boundary conditions defined on its boundaries. Homogenization requires excess pressures that are periodic plus a linear trend, *i.e.*, $p_\alpha(\underline{x}) = \chi_\alpha(\underline{x}) - \underline{H}_\alpha \cdot \underline{x}$ with periodic $\chi_\alpha(\underline{x})$ and constant \underline{H}_α . Then the capillary pressure is equal to a periodic function plus a linear trend, *i.e.*, $\psi(\underline{x}, s_1) = p(\underline{x}, s_1) - \underline{C} \cdot \underline{x}$ with periodic $p(\underline{x}, s_1)$ and constant \underline{C} . Hence, the boundary conditions are such that $\underline{H}_1 - \underline{H}_2 = (\rho_1 - \rho_2) \underline{g} - \underline{C}$.

Since the boundary conditions are specified for excess pressure *differences*, the excess pressures are computed up to an arbitrary constant. Therefore, it makes sense to define the excess pressure difference

$$p_{21} = p_2 - p_1 + \langle p_1 \rangle - \langle p_2 \rangle + P \quad (10a)$$

where $P = \langle p_{21} \rangle$ is a specified *constant*. Now this excess pressure difference is equated to the periodic part of the capillary pressure

$$p(\underline{x}, s_1) = p_{21} \quad (10.b)$$

Equations (10) make it possible to compute the saturations s_α and, hence, the coarse-scale saturations $S_\alpha = \langle \phi s_\alpha \rangle / \langle \phi \rangle$ that belong to the specified periodic part of the coarse-scale capillary pressure $\langle p_{21} \rangle = P$. The determination of the periodic part of the coarse-scale capillary pressure function $P(S_1)$ is considered as one of the most essential parts of the process-independent homogenization approach. At the same time, based upon the fine-scale excess pressure solutions, the coarse-scale mobilities can be determined by the conventional numerical homogenization technique.

In reservoir studies the much simpler approximation $\nabla p_1 = \nabla p_2$ (the ‘no-capillary-gradient case’) is often sufficiently accurate. Especially around a well, where the excess pressure gradients are generally large, this approximation may be useful.

The no-capillary-gradient case plays also an important role in the numerical algorithm to solve the excess pressure equations. If $\nabla p_1 = \nabla p_2$, solutions of the steady equations (9) exist only if $\nu \mu_1 \lambda_1 = \mu_2 \lambda_2$ for any specified constant $\nu \geq 0$. (The viscosities are introduced to normalize the parameter ν .) This constraint can be satisfied only if $\mu_1 \lambda_1 = r_1 \mathbf{k}$ and $\mu_2 \lambda_2 = r_2 \mathbf{k}$. Then steady solutions for the zero-capillary-gradient case exist only if

$$r_2 / r_1 = \nu \quad (\nu \text{ is a specified constant}) \quad (11)$$

Based on relationship (11) a numerical algorithm is developed.

The *computational procedure*, written schematically, is the following :

Step 1 :

- for a given value of ν solve equation (11). As a result a fine-scale distribution of the saturation is obtained.
- Solve equations (9) with appropriate periodic boundary conditions; as a result a fine-scale pressure distribution is obtained. Note that because of condition (11) the pressure fields for the two phases are equal.
- Compute the reference value P of formula (10a) based (for instance) on the analytical *fine-scale* relationship linking the capillary pressure and the saturation. Note that the value of P is directly related to the value of the constant ν .
- If a ‘no-capillary’ case is considered : compute the upscaled value of the saturation S (volume-averaging) and the upscaled values of mobilities (following formulas (3)).

Otherwise perform Step 2 :

- With the local saturation set while performing the ‘no-capillary’ computations, solve equations (9).
- Compute new values of the local capillary pressure based on formula (10a).
- Using the analytical *fine-scale* relationship (10b) linking the capillary pressure and the saturation, compute new approximation of the saturation.

Repeat Step 2 until convergence is reached.

Step 3 :

- compute the averaged values of the saturation and capillary pressure (by volume averaging) and evaluate the components of the coarse-scale mobility tensors (following formulas (3)).

This way for a given v , a pair of the upscaled values of the saturation S and capillary pressures P^c , as well as upscaled values of the mobilities, are evaluated. By repeating the computations for different v , the coarse-scale $P^c - S - K^r$ curves are established.

5. Numerical experiments

In this section the results of numerical experiments are given. As is evident from all the computational procedures given above, the solution of pressure equation of form (2), (4) or, respectively, (9) is the ‘kernel’ of the upscaling procedure. In order to discretise the pressure equation a conformal-nodal finite element method was used. The computational domain was divided into 160000 tetrahedra with 35301 nodes ($41 \times 41 \times 21$).

Consider a periodic heterogeneous porous medium consisting of two materials, and the heterogeneity pattern sketched in Fig. 1. As a periodicity cell a domain of Fig. 2 may be chosen, its dimensions are $\Delta x = \Delta y = 10$ m and $\Delta z = 5$ m. Let us also assume that parts of the medium printed in dark color are of a high permeability ($\mathbf{k}(\underline{x})=10000$ mD), whereas the bright parts are less permeable ($\mathbf{k}(\underline{x})=100$ mD), ($10^{-3}\mu\text{m}^2 = 1.013250$ mD). Thus there exist ‘preferable flow paths’ in a medium; this is demonstrated in Fig. 3 where the streamlines for velocities greater than a given (small) value have been depicted.

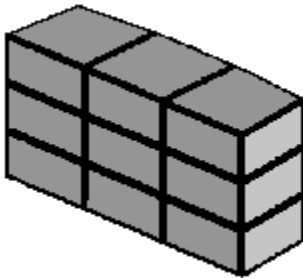


Fig.1 Fine-scale permeability distribution

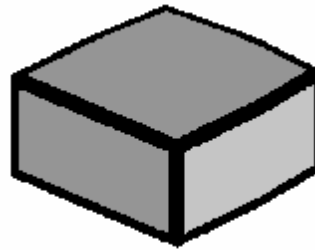


Fig. 2 Homogenization cell

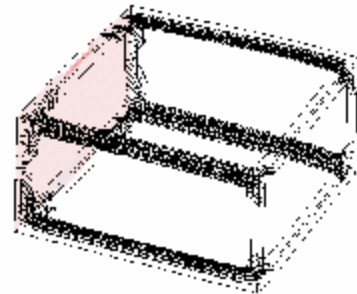


Fig. 3 Preferable flow pattern

The upscaled absolute permeability for such permeability pattern is equal to:

$$\mathbf{K} = \begin{pmatrix} 313.8539 & 0 & 0 \\ 0 & 313.8539 & 0 \\ 0 & 0 & 218.1415 \end{pmatrix} \text{mD.}$$

Now a flow in near-well region is considered, with the heterogeneity pattern as shown in Fig. 4. A cylindrical homogenization cell of a volume $\Delta r \times \Delta \theta \times \Delta z = 10 \text{ m} \times \pi / 4 \text{ m} \times 5 \text{ m}$ directly around a well of a radius 0.25m is considered, Fig. 5.

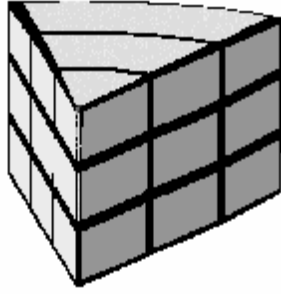


Fig. 4 Permeability distribution in segment around well

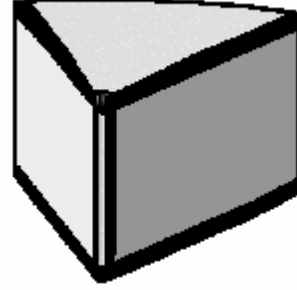


Fig. 5 Cylindrical homogenization cell

Homogenization yields

$$\mathbf{K} = \begin{pmatrix} 402.6744 & 0 & 0 \\ 0 & 723.3713 & 0 \\ 0 & 0 & 225.1438 \end{pmatrix} \text{mD.}$$

Let us now consider a two-phase flow in a domain with heterogeneity pattern as depicted in Fig. 2. The water has viscosity $\mu_1 = 5 \times 10^{-4}$ Pa.s while the oil has viscosity $\mu_2 = 5 \times 10^{-2}$ Pa.s. Let us consider a residual oil saturation of $s_{2R} = 1/5$ and a maximum oil saturation of $s_{2M} = 9/10$. Then the admissible interval for the water saturation is $1/10 \leq s_1 \leq 4/5$.

The relative permeability functions are

$$r_1(s_1) = 2/3 \cdot (s_1^2 - (1 - s_{2M})^2), \quad r_2(s_2) = 8/7 \cdot (s_2^2 - s_{2R}^2).$$

The capillary pressure is given by

$$\psi = p = \tan\left(\frac{s_2 + (s_{2R} + s_{2M})/2}{s_{2M} - s_{2R}} \pi\right) + d$$

where $d = d(\underline{x}) = 5/\mathbf{k}(\underline{x})/2$, which makes the capillary pressure function heterogeneous. Since d is periodic, the capillary pressure ψ is equal to the periodic part of the capillary pressure, p .

Computations were performed for 16 different values of parameter ν varying within the range from 0.0005 to 10000. The results are summarized in Fig. 6 and Fig. 7, where the coarse-scale capillary pressure P^c and coarse-scale mobilities are depicted in function of coarse-scale water saturation S . Because of the heterogeneity pattern of the sample, the resulting coarse-scale mobilities components in x and y directions are equal and are reported only once.

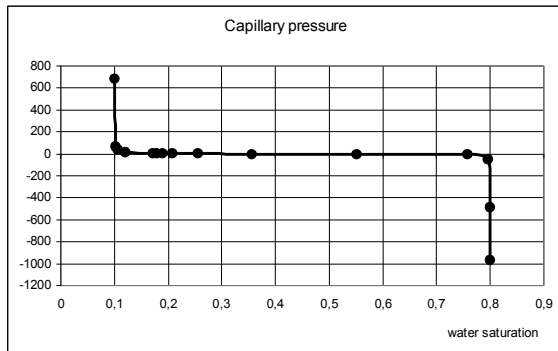


Fig. 6 Coarse-scale capillary pressure in function of coarse-scale saturation.

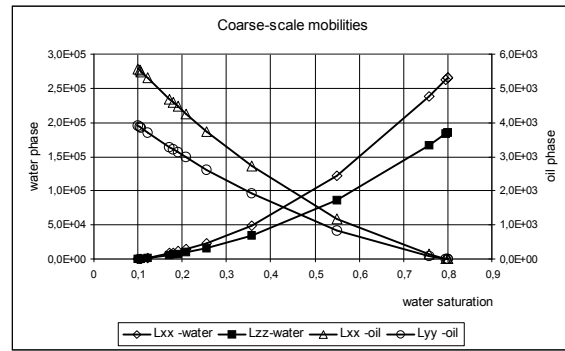


Fig. 7. Coarse-scale mobilities for water and oil in x and z directions.

The fine-scale equations are solved numerically with the finite element method. However, any computational method to numerically solve partial differential equations would be appropriate. With one remark: there is an influence of the method on the final solution by which we mean the upscaled parameter. The conformal-nodal finite element method that has been used gives upper bounds for the coarse-scale mobilities. If information about the accuracy of the homogenization is required, also the mixed-hybrid finite element method can be invoked. This method gives lower bounds for the numerically computed coarse-scale mobilities (Trykozko *et al.* 2001).

7. Summary

To be able to use a limited number of relatively large grid cells in numerical groundwater models and oil reservoir simulators, upscaling of the flow parameters is performed.

In this paper a unified approach was presented for upscaling of flow parameters. The approach is based on the principle of preservation of form stating that the mathematical form of the fine-scale equations should be preserved as much as possible on the coarse scale. Using this principle the range of applicability of homogenization can be made much wider and cover a class of nonlinear problems too. In this paper extensions of the classical homogenization approach to the two-phase flow and flow in near well regions are discussed.

The presented methods belong to a class of process-independent methods. The approach entails the numerical solution of a local fine-scale steady-state flow problems; here the finite element method was used. The results of three numerical experiments are presented.

Homogenization is an upscaling method that has been developed especially for periodic media. Boundary conditions that are consistent with the actual flow might appear superior above the more-or-less arbitrarily chosen periodic boundary conditions. If more accuracy is required, the computationally more expensive global upscaling in which the permeability distribution in the neighbourhood of the upscaling cell is taken into account might be an alternative. However, in that case there is an ambiguity in what is exactly meant by “upscaling”, because pressure-flux averaging (3a) and pressure dissipation averaging (3b) yield different results that cannot be used consistently together (Zijl, Trykozko, 2001a).

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